LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2024**

UMT 5503 – DISCRETE MATHEMATICS

Date: 14-11-2024 Dept. No. Max. : 100 Marks

Time: 09:00 am-12:00 pm

SECTION A

Answer ANY FOUR of the following **4 x 10 = 40 Marks**

1. Prove that $(P \to (Q \to R)) \Rightarrow ((P \to Q) \to (P \to R))$ by constructing the truth table.
2. Obtain the CNF and DNF of $P \to ((P \to Q) \wedge \neg(\neg Q \vee \neg P))$.
3. Examine whether the premises $\neg P \vee Q, R \to S, \neg Q \vee R \Longrightarrow P \to S$.
4. Let R be a set of all real numbers with binary operation ¿ defined by $x * y = x + y + 2xy$ for all $x, y \in R$. Check (i) ¿ is a monoid or not (ii) Is it commutative?
5. Let ¿ be a semigroup and R be a congruence relation on ¿. Then prove that the quotient set S/R is a semigroup $(S/R, \bigoplus)$ where the operation $\bigoplus$ corresponds to the operation ¿ on S. Also show that there exists a homomorphism from ¿ onto $(S/R, \bigoplus)$.
6. State and prove Isotonicity property of lattice.
7. In a complemented distributive lattice, show that the following are equivalent:
   $a \le b \Longleftrightarrow a \wedge b' = 0 \Longleftrightarrow a' \vee b = 1 \Longleftrightarrow b' \le a'$.
8. Prove that in a complemented distributive lattice, complement is unique.

SECTION B

Answer ANY THREE of the following **3 x 20 = 60 Marks**

9. (a) Show that $(\neg P \wedge (\neg Q \wedge R)) \vee ((Q \wedge R) \vee (P \wedge R)) \Longleftrightarrow R$

   (b) Verify that the statement formula $(\neg Q \wedge (P \to Q)) \to \neg P$ is a tautology or not. (10+10)

10. Obtain PDNF and PCNF for the following:

    (i) $(\urcorner P \to R) \wedge (Q \rightleftarrows P)$ (ii) $(P \wedge R) \vee (P \wedge \urcorner Q)$.

11. (a) Check the following set of premises are inconsistent.
    (i) If Tharun gets his degree, he will go for a job.
    (ii) If he goes for a job, he will get married soon.
    (iii) If he goes for higher study, he will not get married.
    (iv) Tharun gets his degree and goes for higher study.

    (b) Using indirect method of proof, prove that $P \to R, Q \to S, P \vee Q \Rightarrow S \vee R$. (10+10)

12. (a) Show that the composition of a semigroup homomorphism is also a semigroup homomorphism.

    (b) Prove that for any commutative monoid ¿, the set of idempotent elements of M forms a sub monoid. (10+10)

13. State and prove the basic properties of lattice. (20)

14. (a) Formulate the product-of-sums canonical forms of the Boolean expression in three variables x_1, x_2, x_3
    (i) $x_1 * x_2$
    (ii) $x_1 \bigoplus x_2$
    (iii) $(x_1 * x_2)' \bigoplus x_3$.

    (b) State and prove De Morgan's law of Boolean Algebra. (10+10)

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